Maximal subgraph enumeration

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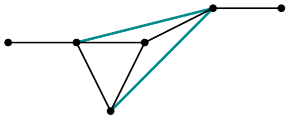
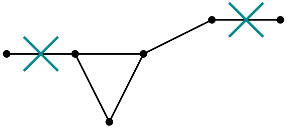
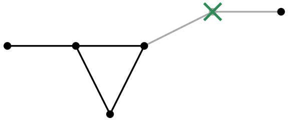
Completions, deletions, induced subgraphs



Completion



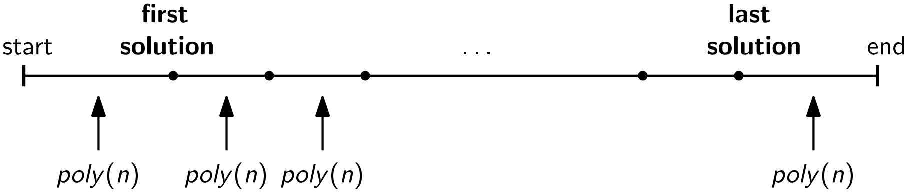
Deletion



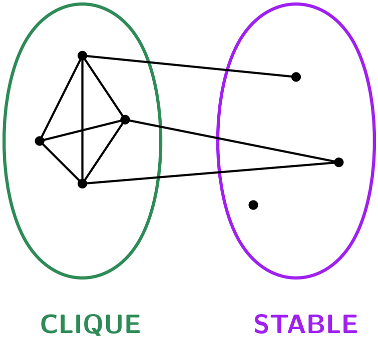
Induced subgraph

Enumeration: Complexity

Input of size n, N solutions to output.



Enumeration of split graphs



The class is hereditary and self-complementary.

Enumeration of split graphs

Theorem (B. *et al.*, 2019)

Enumeration of maximal induced split subgraphs can be done in

- *delay-polynomial time;*
- *polynomial space,*

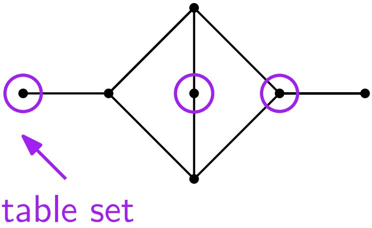
using the restricted problem of [Cohen et al., 2008].

Theorem (B. *et al.*, 2019)

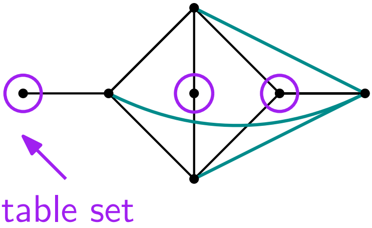
Enumeration of minimal split completions can be done in

- *delay-polynomial time;*
- *polynomial space.*

Minimal split completions: technique

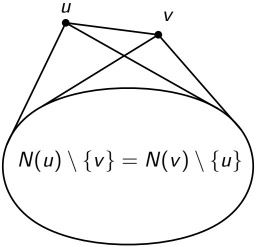


Minimal split completions: technique



If S is not maximal, the induced completion is not minimal.

True twins and collisions



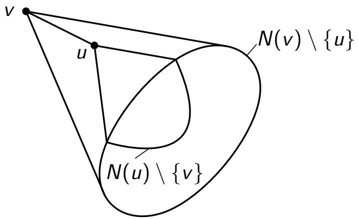
Theorem (Collisions)

Two maximal stable sets S_1 and S_2 induce the same split completion of G if there exists two vertices $x_1 \in S_1$ and $x_2 \in S_2$ such that $S_1 = (S_2 \setminus \{x_2\}) \cup \{x_1\}$ and $S_2 = (S_1 \setminus \{x_1\}) \cup \{x_2\}$ with $N_G[x_1] = N_G[x_2] = V \setminus (S_1 \cap S_2)$.

Redundant vertices

Definition

The vertex v is redundant if there exists another vertex u such that $N[u] \subsetneq N[v]$.



Bijection

Theorem

Let G be a graph, let I be a maximal stable set of G. The split completion induced by I is minimal if and only if I does not contain a redundant vertex x such that $V \setminus N(x)$ *is a stable set.*

$$\left\{ \begin{array}{c} \text{Minimal split} \\ \text{completions of } G \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \text{Maximal stable sets} \\ \text{of a modified graph} \end{array} \right\}$$

Enumeration of minimal split completions

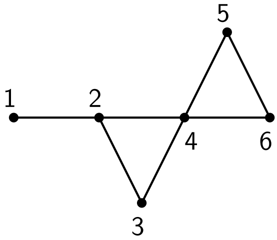
- Avoid non-minimal split completions:
  - maximal stable sets;
  - redundant vertices x such that $V \setminus \{x\}$ is a stable set;
- Avoid collisions:
  - true twins: same as redundant.

Algorithm

Algorithm

Input: *a graph G.*

Output: *all minimal split completions of G.*



1		
2		
3		
4		
5		
6		

Algorithm

Algorithm

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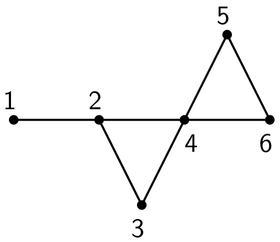


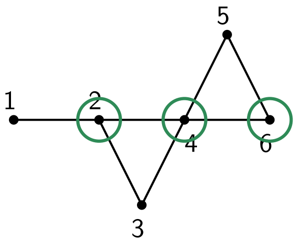
1	$\{1,2\}$	
2	$\{1,2,3,4\}$	
3	$\{2,3,4\}$	
4	$\{2,3,4,5,6\}$	
5	$\{4,5,6\}$	
6	$\{4,5,6\}$	

Algorithm

Algorithm

Input: *a graph G.*

Output: *all minimal split completions of G.*



1	$\{1,2\}$	
2	$\{1,2,3,4\}$	×
3	$\{2,3,4\}$	
4	$\{2,3,4,5,6\}$	×
5	$\{4,5,6\}$	
6	$\{4,5,6\}$	×

Algorithm

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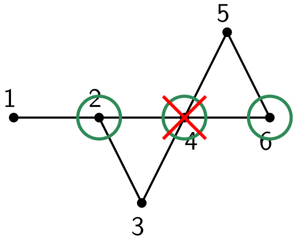

1	$\{1,2\}$	
2	$\{1,2,3,4\}$	×
3	$\{2,3,4\}$	
~~4~~	~~$\{2,3,4,5,6\}$~~	~~×~~
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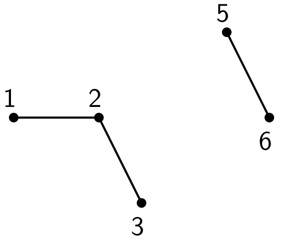

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2	$\{1, 2, 3, 4\}$	×
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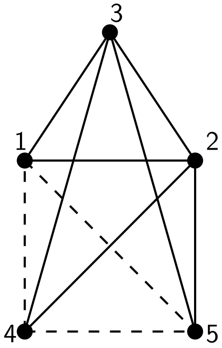
~~4~~	~~$\{2, 3, 4, 5, 6\}$~~	~~×~~
5	$\{4, 5, 6\}$	
6	$\{4, 5, 6\}$	×

Enumerate maximal stable sets in time delay-polynomial and polynomial space (reverse search [Avis and Fukuda, 1996]).

Enumeration of cographs

Stable under complement and disjoint union; also known as P_4-free graphs.



The class is hereditary and self-complementary.

Enumeration of cographs

Theorem (B. *et al.*, 2019)

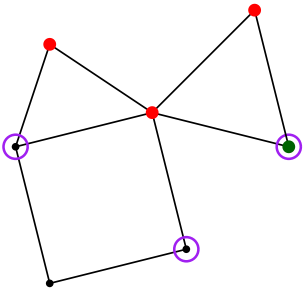
The enumeration of maximal induced sub-cographs and minimal cograph deletions (equivalently completions) can be done in

- *delay-polynomial time;*
- *exponential space,*

using proximity search [Conte and Uno, 2019].

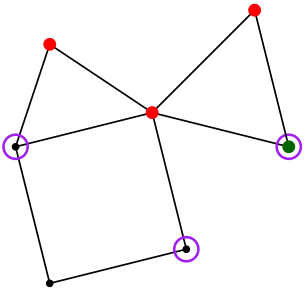
Extension problem

Given two disjoints subsets A and $B \subseteq V$, does there exist a maximal induced subgraph containing A and avoiding B with the desired property?



Extension problem

Given two disjoints subsets A and $B \subseteq V$, does there exist a maximal induced subgraph containing A and avoiding B with the desired property?



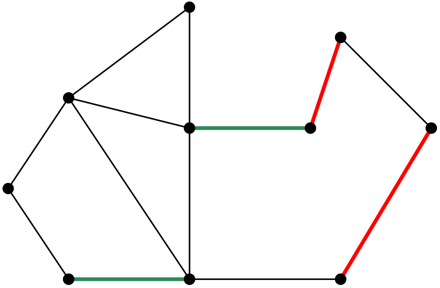
Theorem (B. *et al.*, 2019)

The extension problem for nontrivial hereditary properties is NP-complete.

In the fashion of [Lewis and Yannakakis, 1980].

Perspectives

Extension problem: What about the edge version?



- Triangle-free graphs
- Planar graphs
- Generic algorithm

Thank you!