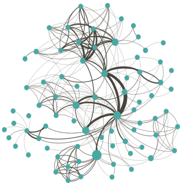
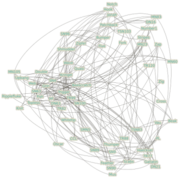
Structural Rounding

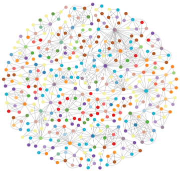
Brian Lavallee

23 January 2020

Real World Networks







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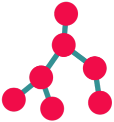

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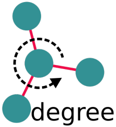

degree d

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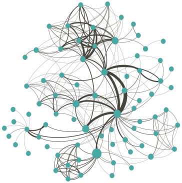

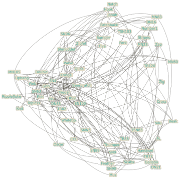

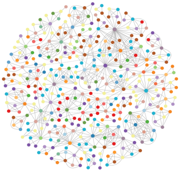

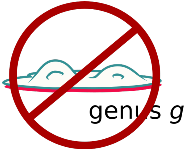

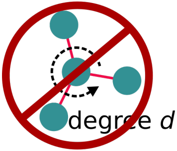

Structural Rounding

Edit: Use φ to edit G to be in $\mathcal{C}_\lambda$. A problem is **stable** wrt φ with constant c' if $\mathrm{OPT}(G') < \mathrm{OPT}(G) + c' \cdot d$ for any d-editable G'.

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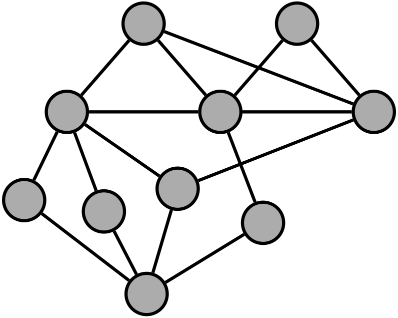
Theorem (Demaine et al, 2019)

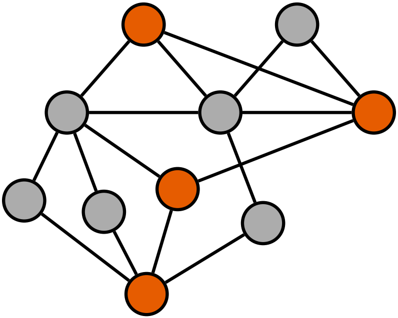
Let π be a problem that is stable under φ with constant c' and that can be structurally lifted wrt φ with constant c. If π has a polynomial-time $\rho(\lambda)$-approximation algorithm in the graph class $\mathcal{C}_\lambda$, and $(\mathcal{C}_\lambda, \varphi)$-EDIT has a polynomial-time (α, β)-approximation algorithm, then there is a polynomial-time $((1 + c'\alpha\delta) \cdot \rho(\beta\lambda) + c\alpha\delta)$-approximation algorithm for π on graphs that are $(\delta \cdot \mathrm{OPT}_\pi(G))$-close to $\mathcal{C}_\lambda$.

Structural Rounding Example



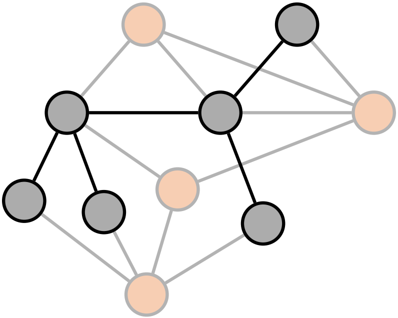
Initial graph G.

Structural Rounding Example: Edit



Edit G to have some nice property.

Structural Rounding Example: Edit



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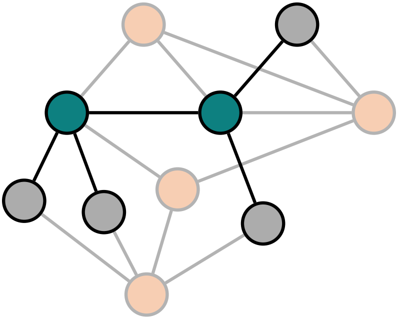
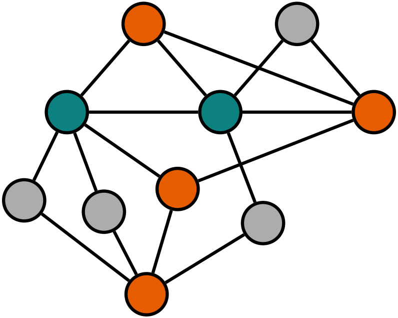
Structural Rounding Example: Solve



Solve on the edited graph.

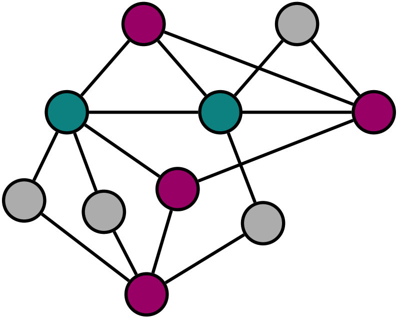
Structural Rounding Example: Lift



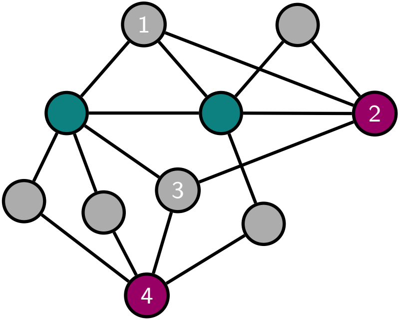
Lift the solution back to G.

Structural Rounding Example: Lift

Naïve Lifting



Greedy Lifting



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Structural Rounding Theorem

Theorem (Demaine et al, 2019)

Let π be a problem

- *that is stable under φ with constant c' and*
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*If π has a poly-time $\rho(\lambda)$-approximation in $\mathcal{C}_\lambda$ and $(\mathcal{C}_\lambda, \varphi)$-*EDIT *has a poly-time (α, β)-approximation, then there is a poly-time $((1 + c'\alpha\delta) \cdot \rho(\beta\lambda) + c\alpha\delta)$-approximation for π on graphs that are $(\delta \cdot \mathrm{OPT}_\pi(G))$-close to $\mathcal{C}_\lambda$.*

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Lift: VertexCover can be structurally lifted wrt vertex deletion with $c = 1$. Naïve approach just adds every edit to the solution. **Greedy approach does a bit better, but still feels wasteful.**

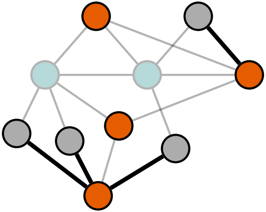
Better Lifting

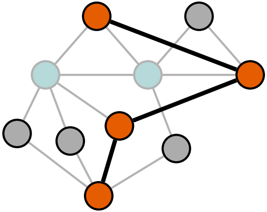
Vertex Cover Lift

Input: Graph $G = (V, E)$, an edit set $\mathbf{D} \subseteq V$, and a vertex cover $\mathbf{C}$ of $G[V \setminus \mathbf{D}]$

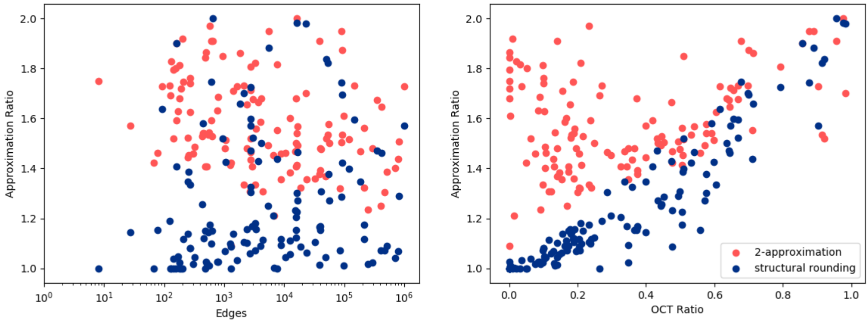
Question: What is the minimum size of a set $\mathbf{L} \subseteq V \setminus \mathbf{C}$ such that $\mathbf{L} \cup \mathbf{C}$ is a vertex cover of G?

algorithm	2apx	oct-first	bip-first
approx. ratio	2	$\frac{2}{1+p}$	$\frac{1}{p}$





Results



algorithm	`dfs`	`standard`	`sr-greedy`	`sr-oct-first`
runtime 4M	2.418	1.511	3.613	4.521

Editing Algorithms

Graph Family $\mathcal{C}_\lambda$	Edit Operation φ	
	Vertex Deletion	**Edge Deletion**
Bounded Degeneracy (r)	$O(r \log n)$-approx. $\left(\frac{4m-\beta rn}{m-rn}, \beta\right)$-approx. $\left(\frac{1}{\epsilon}, \frac{4}{1-2\epsilon}\right)$-approx. ($\epsilon < 1/2$) $o(\log(n/r))$-inapprox.	$O(r \log n)$-approx. – $\left(\frac{1}{\epsilon}, \frac{4}{1-\epsilon}\right)$-approx. ($\epsilon < 1$) $o(\log(n/r))$-inapprox.
Bounded Treewidth (w)	$(O(\log^{1.5} n),\ O(\sqrt{\log w}))$-approx. $o(\log n)$-inapprox. for $w \in \Omega(n^{1/2})$	$(O(\log n \log\log n),\ O(\log w))$-approx.[1] –

[1] Bansal et al, 2017

Thanks

Repo: `github.com/TheoryInPractice/structural-rounding`

- Erik D. Demaine
- Timothy D. Goodrich
- Kyle Kloster
- Quanquan C. Liu
- Hayley Russell
- Blair D. Sullivan
- Ali Vakilian
- Andrew van der Poel

We're Hiring!

Looking for Ph.D. students and Postdocs with an interest in theory in practice. Contact Blair Sullivan at sullivan@cs.utah.edu.

OCT Heuristic