Switching checkerboards

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We denote by $A(G) = (a_{ij})_{1\leq i,j\leq n}$ the adjacency matrix of a graph, λ_1 its largest eigenvalue, $M_1(G) = \sum_{i\in V} d_i^2$ and $M_2(G) = \sum_{ij\in E} d_i d_j$ the first two Zagreb indices of G where d_i is the degree of vertex i. A *positive* (resp. *negative*) checkerboard is a set of four distinct coordinates i, j, k, l with $i < j$ and $k < l$ such that $a_{ik} = a_{jl} = 1$ (resp. 0) and $a_{il} = a_{jk} = 0$ (resp. 1). *Switching* a checkerboard means changing it from positive to negative or vice versa. Switching checkerboards does not change the degree distribution. We are interested in the impact of switching checkerboards on both the eigenvalues and $M_2(G)$. Let $\mathcal{M}(\mathcal{D})$ be the set of adjacency matrices of simple graphs with row sums given by a degree distribution $\mathcal{D}$, we denote by $G(\mathcal{D})$ the oriented graph with vertex set $\mathcal{M}(D)$ and an arc from A to A' if A' can be obtained from A by switching a negative checkerboard. We proof that for any degree distribution $\mathcal{D}$, there exists in $\mathcal{M}(\mathcal{D})$ a matrix without negative checkerboards which maximises the spectral radius over $\mathcal{M}(\mathcal{D})$ and give some properties of $G(\mathcal{D})$ in terms of threshold graphs. We analyse the dynamics of $M_2(G)$ and $\lambda_1(G)$ along the edges of $G(\mathcal{D})$ and revisit a problem by Nikiforov linking λ_1^2 and $M_2(G)$.