Enumeration of minimal completions and maximal induced subgraphs*

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Given a graph G and a property Π, we are interested in all (inclusion-wise) minimal completions and deletions of G into a graph that has property Π. We are also interested in another related problem: finding all maximal induced subgraphs of G which fulfill property Π. These problems are treated from the viewpoint of enumeration. We are looking for algorithms in *polynomial-delay* time complexity to enumerate minimal completions and deletions, and maximal induced subgraphs.

Split graphs are graphs whose vertex set can be partitioned into a clique and a stable set. They have been widely studied in [3] in order to find efficiently one minimal completion. Establishing a bijection between the minimal split completions of a graph and the maximal stable sets of a modified graph, we show that there exists a polynomial-delay algorithm for the enumeration of all minimal split completions of a given graph. As the class of split graphs is stable under complementation, the same holds for minimal deletions. Then, using the enumeration technique presented in [1], we show that the same complexity can also be obtained for the enumeration of maximal induced split subgraphs.

We obtain similar results for the class of cographs, namely the existence of a delay-polynomial algorithm for the enumeration of all minimal cograph deletions (and completions, since the class is self-complementary), and for the enumeration of maximal induced sub-cographs. These results rely on the *proximity search* algorithm recently introduced in [2].

Another problem of interest, as its resolution would imply a polynomial-delay algorithm for the enumeration of maximal induced Π-sugraphs for any property Π, is the *extension problem*: given G, and A, $B \subseteq V(G)$, does there exist a maximal induced Π-subgraph containing A and avoiding B? With a proof similar to [4], it turns out that this problem is also NP-complete for any hereditary and nontrivial property.

References

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