Structured Connectivity Augmentation

Fedor V. Fomin[*]   Petr A. Golovach[*]   Dimitrios M. Thilikos[†]

We initiate the algorithmic study of the following "structured augmentation" question: is it possible to increase the connectivity of a given graph G by superposing it with another given graph H? More precisely, graph F is the superposition of G and H with respect to injective mapping $\varphi : V(H) \rightarrow V(G)$ if every edge uv of F is either an edge of G, or $\phi^{-1}(u)\phi^{-1}(v)$ is an edge of H. We consider the following optimization problem. Given graphs G, H, and a weight function ω assigning non-negative weights to pairs of vertices of $V(G)$, the task is to find φ of minimum weight $\omega(\phi) = \sum_{xy \in E(H)} \omega(\phi(x)\varphi(y))$ such that the edge connectivity of the superposition F of G and H with respect to φ is higher than the edge connectivity of G. Our main result is the following "dichotomy" complexity classification. We say that a class of graphs $\mathcal{C}$ has bounded vertex-cover number, if there is a constant t depending on $\mathcal{C}$ only such that the vertex-cover number of every graph from $\mathcal{C}$ does not exceed t. We show that for every class of graphs $\mathcal{C}$ with bounded vertex-cover number, the problems of superposing into a connected graph F and to 2-edge connected graph F, are solvable in polynomial time when $H \in \mathcal{C}$. On the other hand, for any hereditary class $\mathcal{C}$ with unbounded vertex-cover number, both problems are NP-hard when $H \in \mathcal{C}$. For the unweighted variants of structured augmentation problems, i.e. the problems where the task is to identify whether there is a superposition of graphs of required connectivity, we provide necessary and sufficient combinatorial conditions on the existence of such superpositions. These conditions imply polynomial time algorithms solving the unweighted variants of the problems.

[*]Department of Informatics, University of Bergen, Norway.
[†]AlGCo project team, CNRS, LIRMM, Université de Montpellier, Montpellier, France.