Building Large k-cores in Graphs of Small Degeneracy

Abstract

The k-core in a graph G is the maximal induced subgraph of G with all vertex degrees at least k. For a given graph G and integers k, b and p, is it possible to add at most b edges to G so that there is a k-core with at least p vertices? This problem is motivated by the studies of unraveling phenomena in social networks and was introduced by Chitnis and Talmon under the name Edge k-Core. We give polynomial/FPT algorithms for Edge k-Core on several classes of low degeneracy graphs: forests, graphs of small vertex cover, and graphs of small treewidth.

While each of our three algorithms is based on completely different approaches, each of them is built upon a nice graph-theoretical result. For example, the polynomial time algorithm on forests strongly exploits the theorem of Henning and Yeo (J. Graph Theory 2018) on matchings in graphs of bounded degrees, while the integer linear program in the algorithm for bounded vertex cover requires the celebrated Erdős-Gallai theorem from 1960 about graphic sequences.