Computing Subset Transversals in H-Free Graphs

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For a *vertex deletion problem*, given a graph G the goal is to find a subset of $V(G)$ of size at most k whose deletion from G results in a graph with property π. Vertex deletion problems can also be viewed in the general framework of *graph transversal problems*: can we find a set $V' \subseteq V(G)$ of size at most k such that V' transverses all subsets of $V(G)$ that induce a graph *not* having property π. We consider the subset variant of graph transversal problems where, for some $T \subseteq V(G)$, we only require that V' transverses the vertex subsets of G that do not have property π *and contain at least one vertex of* T. In particular, we are interested in the computational complexity of such problems when the input is restricted to H-free graphs for some fixed graph H; that is, when restricted to the class of graphs that do not contain some fixed graph H as an induced subgraph.

We focus on two particular known problems in this framework, namely Subset Feedback Vertex Set and Subset Odd Cycle Transversal. By combining known and new results we determine the computational complexity of both problems on H-free graphs for every graph H except when $H = sP_1 + P_4$ for some $s \geq 1$.